Broom Barns Community Primary School

Strategies Taught
in School for

Multiplication and
Division

Information for Parents

Multiplication

To multiply successfully, children need to be able to:

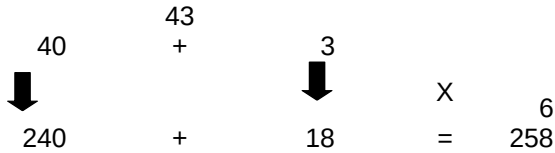
- Recall all multiplication facts to 10 x 10;
- Partition number into multiples of one hundred, ten and one;
- Work out products such as 70 x 5, 70 x 50, 700 x 5 or 700x 50 using the related fact 7 x 5 and their knowledge of place value;
- Add two or more single-digit numbers mentally;
- Add multiples of 10 (such as 60+70) or of 100 (such as 600+700) using the related addition fact, 6+7, and their knowledge of place value;
- Add combinations of whole numbers using the column method.

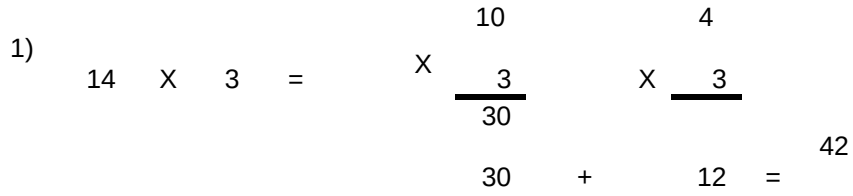
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Stage One: **Mental multiplication using partitioning**

Mental methods for multiplying TU x U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

Informal recording in Year 4 might be:



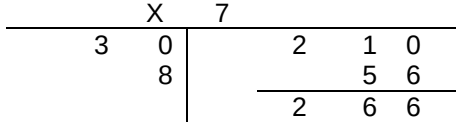


Stage Two : **The grid method**

Step 1

The next step is an expanded method which uses a grid this links directly to the mental method. It is an alternative way of recording the same steps.

It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add. Question; 38X7



3 8 X 7 = 2 6 6

Step 2

following on from this is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.

	0
X	
	2
	2

The grid method may be the main method used by children whose progress is below age related expectations and who have difficulty with mental and written calculation skills.

Stage Three: Expanding short multiplication

The next step is to represent the method of recoding in as column format, but showing the working. Draw attention to the links with the grid method.

	3	8	
	X	7	
2	1	0	(30 X 7)
+	5	6	(8 X 7)
2	6	6	

38 X 7 = 266

Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first steps in 38 x 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 x 7 should be stressed. Most children should be able to use this expanded method for TU X U by the end of year 4.

Stage Four: **Short multiplication**

The recording is reduced further, with carry digits recorded below the line or above.

```
   5
   3   8
 X     7
-------
 2   6   6
```

38 X 7 = 266

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

If, after practice, children cannot use the compact method without making errors, they should return to the expanded format.

Stage Five: **Two digit by two digit products**

Step 1

Extend to TU x TU, asking children to estimate first. Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

		5 0	
2 0	1 0 0	1 2 0	
0 7	3 5 0	4 2	

```
   1 1 2 0
+  0 3 9 2
---------
   1 5 1 2
```

As in the grid method for TU x U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

	50	6	
X	20	7	
	1000	350	1350
	120	42	162
			1512
			1

Step 2

Reduce the recording, showing the links to the grid method.

		5	6	
	X	2	7	
		4	2	6x7
	3	5	0	50x7
	1	2	0	6x20
1	0	0	0	50x20
1	5	1	2	
	1			

Step 3

Reduce the recording further

		5	6	
X		2	7	
	3	9	2	56x7
1	1	2	0	56x20
1	5	1	2	
	1			

The carry digits in the partial products of 56x20=120 and 56x7=392 are usually carried mentally. The aim is for most children to use this long multiplication method for TUxTU by the end of Year 5.

Stage Six: **Three digit by two digit products**

Step 1

Extend to HTU x TU asking children to estimate first. Start with the grid method.

			20	9
2	0	0	40000	1800
	8	0	1600	720
		6	120	54

4000
1800
1600
720
120
\+ 54
8294

286 x 29 = 8294

It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

Step 2

Reduce the recording, showing the links to the grid method.

286
X 29

4000	200 x 20 = 4000
1600	80 x 20 = 1600
120	6 x 20 = 120
1800	200 x 9 = 1800
720	80 x 9 = 720
54	6 x 9 = 54
8294	
1	

This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

Step 3

Children who are already secure with multiplication for TU x U and TU x TU should have little difficulty in using the same method for HTU x TU.

286 x 29 is approximately 300x30 = 9000.

	2	8	6	
	X	2	9	
5	**7**	2	0	286 X 20
2	5	7	4	286 X 9)
8	2	9	4	
1				

Again, the carry digits in the partial products are usually carried mentally.

Division

To divide successfully in their heads, children must need to be able to:

- Understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- Recall multiplication and division facts to 10 x 10, recognize multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- Know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- Understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- Understand division as repeated subtraction;
- Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- Multiply a two-digit number by a single-digit number mentally;
- Subtract numbers using the column method.

Stage One: **Mental division using partitioning**

Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.

84	÷	7								
10	X	7	=	70		84	-	70	=	14
2	X	7	=	14						
10	+	2	=	12		**84**	**÷**	**7**	**=**	**12**

Many children can partition and multiply with confidence. But this is not the case for division. It is therefore important that the mental methods of division, such as stressing the correspondence to mental methods of multiplication are given as much attention as learning times table facts, e.g. children need to be as secure in knowing 35 ÷ 7 = 5 as knowing 5 x 7 = 35.

X	?	?	?	?
7	7	0	1	4

84 ÷ 7 = 12

X	1	0	0	2
7	7	0	1	4

Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.

Stage Two: **Short division of TU÷U**

'Short' division of TU÷U can be introduced as a more compact recording of the mental method of partitioning.

For 81÷3, the dividend of 81 is spilt into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60+21. Each number is then divided by 3.

$$
\begin{aligned}
81 \div 3 &= (60 + 21) \div 3 \\
&= (60 \div 3) + (21 \div 3) \\
&= 20 + \mathbf{7} \\
&= 27
\end{aligned}
$$

The short division method is recorded like this:

$$
\begin{array}{r|l}
 & 20 + 7 \\
\hline
3 & 60 + 21
\end{array}
$$

This is shortened to:

$$
\begin{array}{r|l}
 & 27 \\
\hline
3 & \begin{array}{l} 2 \\ 81 \end{array}
\end{array}
$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In this recording it is written in front of the 1 to show that 21 is to e divided by 3. The 27 written above the line represents the answer: 20+7, or 2 tens and 7 ones.

Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. For most children this will be at the end of Year 4 or the beginning of Year 5.

Stage Three: **Expanded method for HTU÷U**

Step 1

This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
For TU÷U there is a link to the mental method.

As you record the division, ask; 'How many nines in 90?' or 'What is 90 divided by 9?' Once they understand and can apply the method, children should be able to move on from TU÷U to HTU÷U quite quickly as the principles are the same.

This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. Chunking is useful for reminding children of the link between division and repeated subtraction.

```
          3   2     r   4
      ┌──────────
    6 │ 1   9   6
      │
      -   6   0     (6 X  10)
      ──────────
      1   3   6
      -   6   0     (6 X  10)
      ──────────
          7   6
      -   6   0     (6 X  10)
      ──────────
          1   6
          1   2     (6 X   2)
      ──────────
                4
```

However, children need to recognize that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.

Step 2
The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU÷U involves multiplying the divisor by multiplies of 10 to find the two multiples that 'trap' the HTU dividend.

To find 196÷6, we start by multiplying 6 by 10, 20, 30, … to find that 6 x 30 = 180 and 6 x 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the number 196 ÷ 6 is between 30 and 40.

Estimating has two purposes when doing a division:

1. to help to choose a staring point for the division;
2. to check the answer after the calculation.

Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.

6	1	9	6	
-	1	8	0	3x30
		1	6	
-		1	2	6x2
			4	32
	Answer:			32R4

The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.

Stage Four: **Short Division of HTU÷U**

'Short' division of HTU÷U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction.

For 291÷3, because 3x90=270 and 3x100=300, we use 270 and split the dividend of 291 into 270 +21. Each part is then divided by 3.

$$
\begin{aligned}
291 \div 3 &= (270+21) \div 3 \\
&= (270 \div 3) + (21 \div 3) \\
&= 90+7 \\
&= 97
\end{aligned}
$$

The short division method is recorded like this:

$$
\begin{array}{r|lll}
 & 0 & 9 & 7 \\
\hline
3 & 2 & 9 & 1 \\
 & 2 & 7 & \\
\hline
 & & 2 & 1 \\
 & & 2 & 1 \\
\hline
 & & & 0
\end{array}
$$

This is then shortened to:

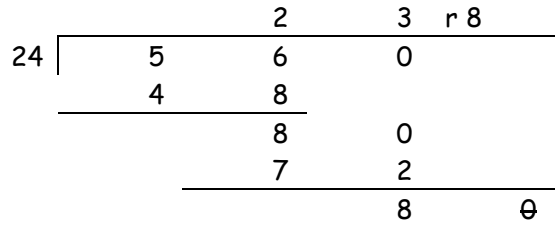
$$
\begin{array}{r|lll}
 & & 9 & 7 \\
\hline
3 & 2 & 9^{2} & 1
\end{array}
$$

Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

For most children this will be at the end of Year 5 or the beginning of Year 6.

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

Can I do long division?



The next steps with division would be thinking about how the remainder part of the quotient is written and the children would learn how to write this quotient as a fraction and then later as a decimal.

Stage Four: **Long Division**

The next step is to tackle HTU ÷ TU, which for most children will be in Year 6.

The layout below, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

24	5	6	0	
20-	4	8	0	24x20
		8	0	
		7	2	24x3
			8	

Therefore 560 ÷ 24 = 23R8

Glossary of mathematical terms

Divisor: a number that divides exactly into another number e.g. 2 is a divisor of 10

Quotient: the number you get by dividing one number by another (the answer)

Product: the answer when two numbers are multiplied

Partitioning: splitting a number into its value, e.g. 235 = 200 + 30 + 5

Distributive Law: multiplying a number is the same as multiplying its addends by the number then adding the products e.g.
6 x 9 is the same as 6 x (4+5)
Which equals (6x4) + (6x5)

Dividend: the number being divided, e.g. 10 ÷ 2 = 5

10 is the dividend

Multiple: a number is added to itself a number of times,

e.g. 4 + 4 + 4 + 4 = 16 4 x 4 = 16