# Progression in Mental Mathematics

A pathway from EYFS to Year 6

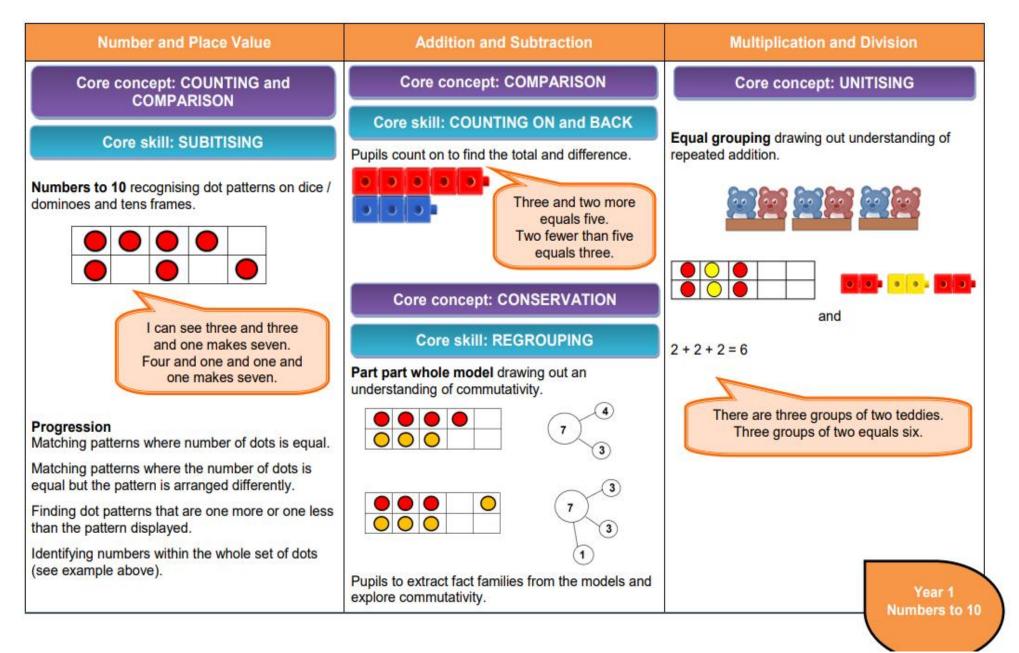


# Nursery

Number and Place Value	Addition and Subtraction	Multiplication
Recite numbers past 5.	Compare two groups of objects, saying	Compare two groups of objects, saying
Develop fast recognition of up to 3	when they have the same number.	when they have the same number.
objects, without having to count them	Show an interest in number problems.	Show an interest in number problems.
individually ('subitising')	Separate a group of three or four	
Say one number for each item in	objects in different ways, beginning to	
order: 1,2,3,4,5.	recognise that the total is still the	
Snow that the last number reached	same.	
when counting a small set of objects	Solve real world mathematical	
tells you how many there are in total	problems with numbers up to 5.	
('cardinal principle').	Compare quantities using language	
Show 'finger numbers' up to 5.	'more than' and 'fewer than'.	
Link numerals and amounts: for		
example, showing the right number of		
objects to match the numeral, up to 5.		
Experiment with their own symbols and marks as well as numerals.		

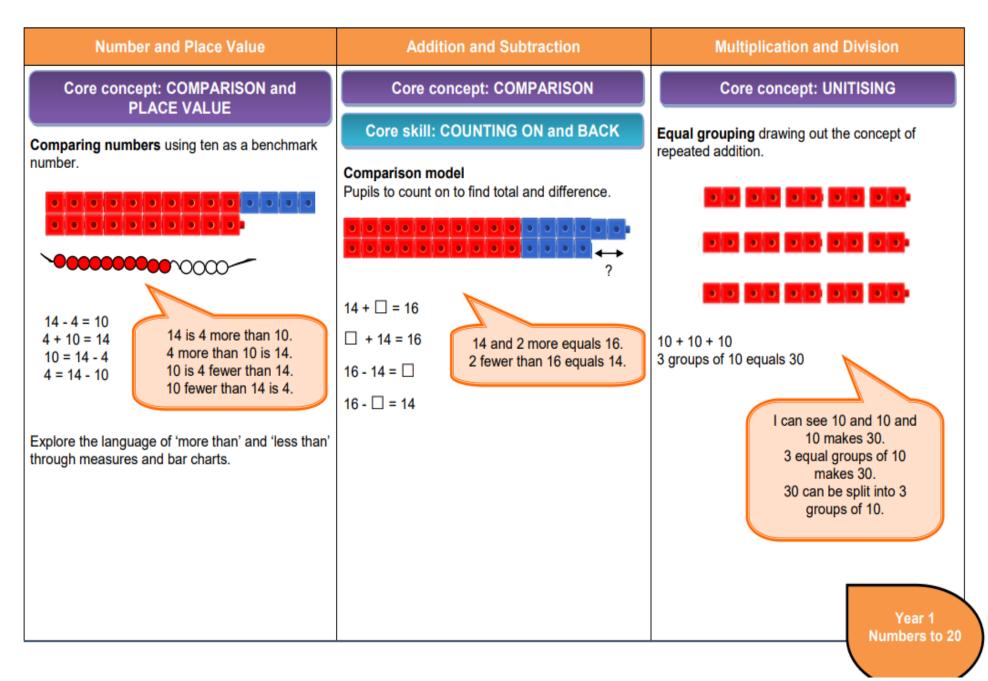
# Reception

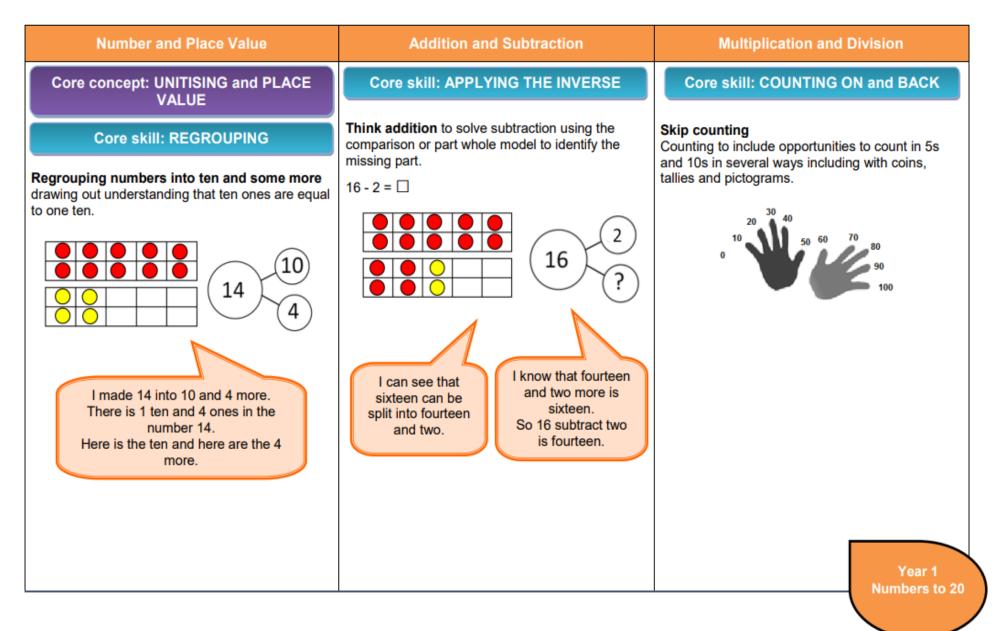
Number and Place Value	Addition and Subtraction	<b>Multiplication and Division</b>
Count objects, actions and sounds.	Use the language of more and fewer to compare two	Begin to identify own
Subitise to 5.	sets of objects.	mathematical problems based
Link the number symbol (numeral) with its cardinal number value.	Find the total number of items in two groups by counting all of them.	on own interests. Represent patterns such as doubles and equal quantities.
Count verbally to 20.	Find one more and one less from a group of up to ten objects.	Solve problems using doubling,
Understand the one more than/ one less than relationship between	Automatically recall number bonds within 10.	halving and sharing.
consecutive numbers.	In practical activities, use the vocabulary involved in	
Explore the composition of numbers to	adding and subtracting.	
10.	Explore and represent patterns within numbers up to	
Have a deep understanding of the composition of the numbers to 10.	10, including evens and odds, double facts and how quantities can be distributed evenly.	
	Record, using marks that they can interpret and explain.	
	Use the language of greater than, less than and the same as with quantities up to 10.	



Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core skill: APPLYING THE INVERSE	Core skill: COUNTING ON and BACK
	<b>Think addition</b> to solve subtraction – leading to greater understanding that if we know one part we can use that to find the unknown part.	<b>Skip counting</b> Counting groups of objects with two hands (drawing out understanding of doubles).
		To include opportunities to count in 2s in several ways.
	I can see that eight can be split into three and five.	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
	I can see that five more than three is eight.	10 10 10 10 10 10 10 10 10 10 10 10 10 1

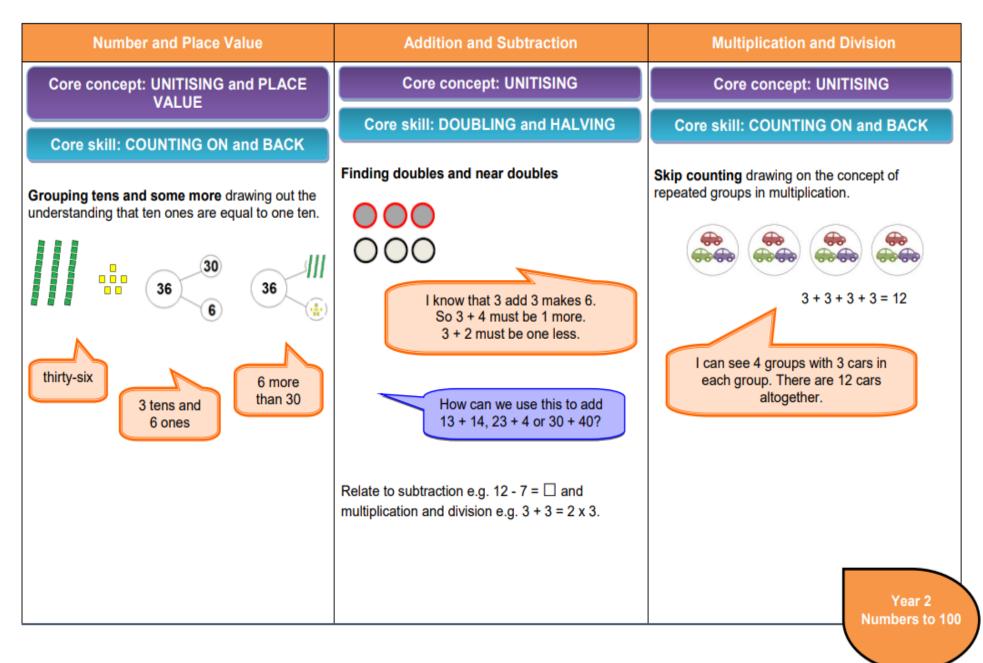
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION	
	Core skill: REGROUPING	
	Think 5 for addition using five as a benchmark number. 4 + 3 =	
	or	
	4+ 3 3+ 4 1 2 2 2	
	4 + 1 + 2 = 3 + 2 + 2 = 5 + 2 = I know that four and one more is five. I can see that three can be split into one and two. Five and two	
	more is seven. equal to seven.	Year 1 Numbers to 10

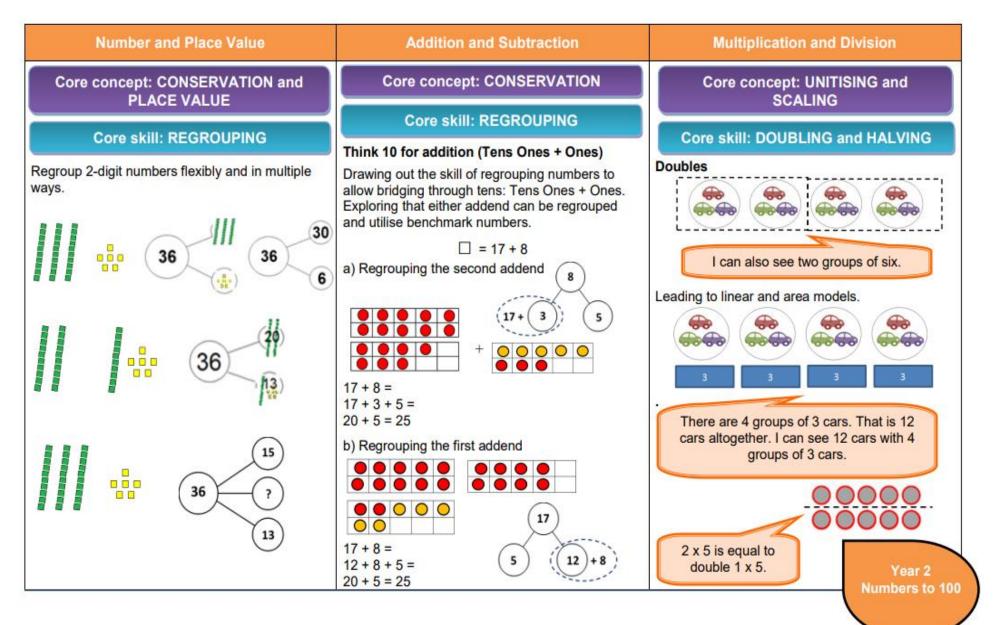




Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION	
	Core skill: REGROUPING	
	Regrouping numbers to 20 leading to ' <b>think 10 for addition</b> '. Pupils should experience regrouping either addend.	
	8+6	
	can become	
	O         O	
	8+2+4= 10+4= 10+4= 6+4+4= 10+4=	
	10+4-	
	I know that eight and two more is ten. Four and two make six. So 8 + 6 can become 8 + 2 + 4. I know that six and four more is ten. Four and four make ten. So 6 + 8 can become 6 + 4 + 4.	Year Numbers

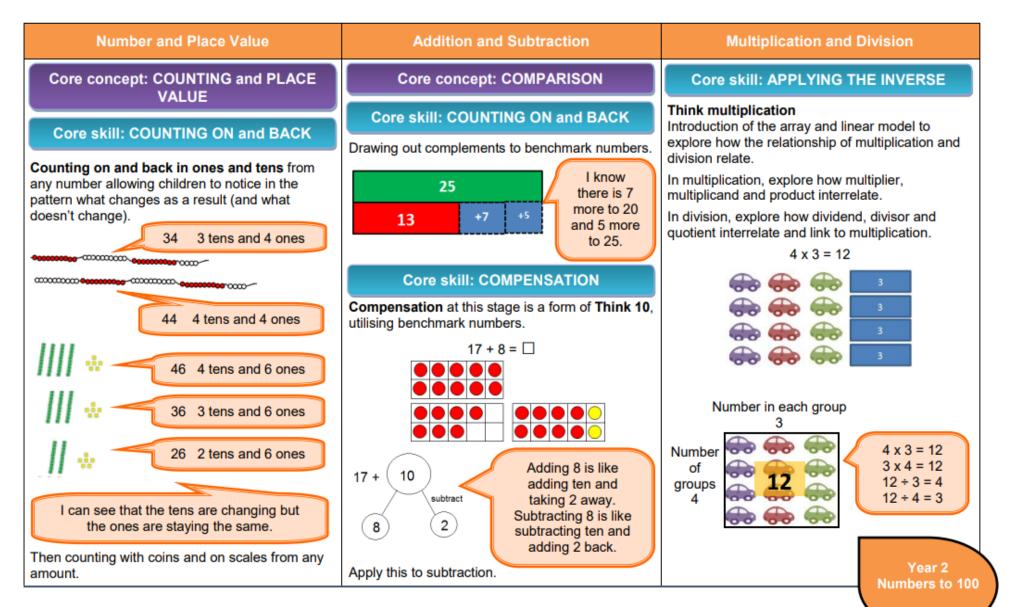
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Then with numbers which would require bridging through ten.	
	For example, 17 - 9	
	Regrouping the subtrahend Regrouping the minuend	
	$ \bigcirc \bigcirc$	
	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
	Nine can be regrouped into 7 and 2. I can take 7 from 17 to leave 10 and then I can use my number bonds to take away 2 more. Seventeen can be regrouped into 10 and 7. Then I can use my number bonds to take 9 from 10. I'm left with 1. Then I add one to seven.	
		Year 1 Numbers to 20
	1	Numbers to 20

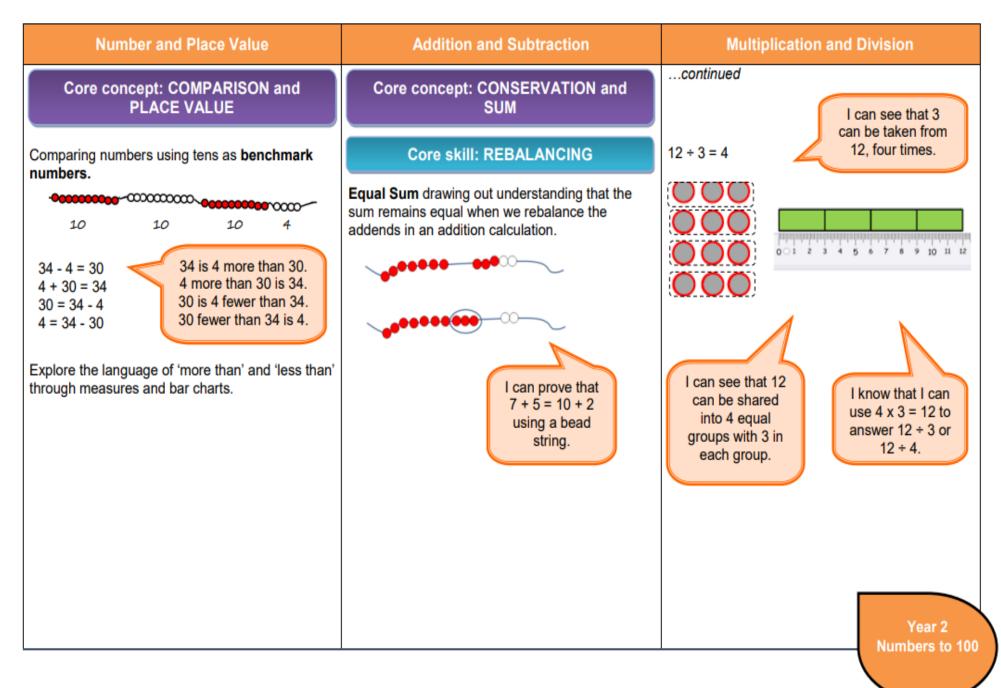


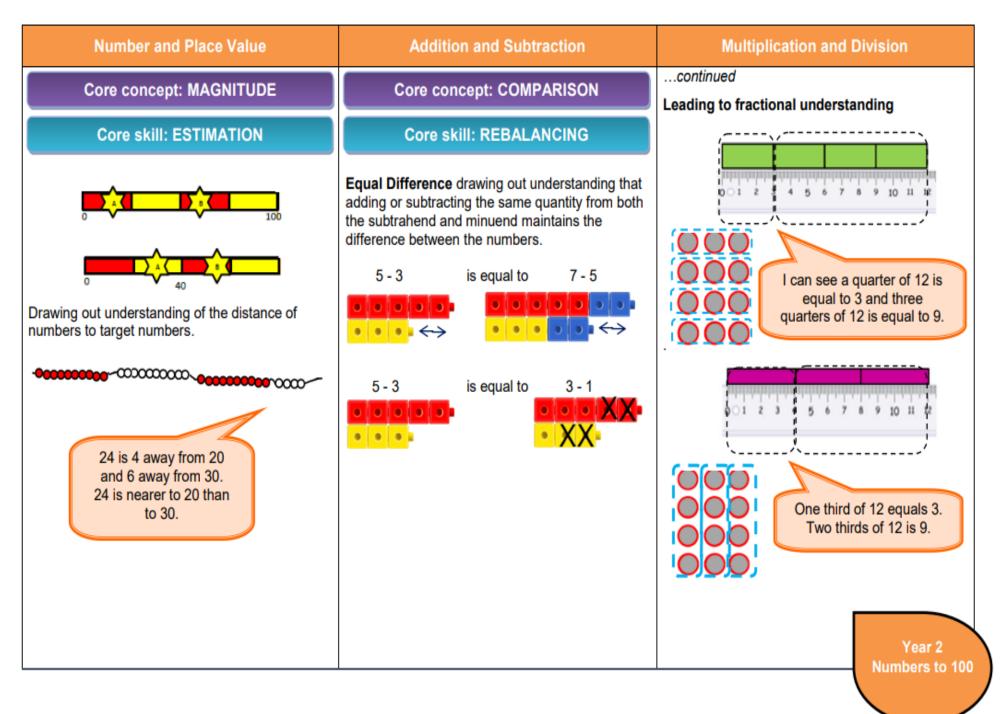


Number and Place Value	Addition and Subtraction	Multiplication and Division
	Think 10 for subtraction (Tens Ones - Ones)         Exploring that either the minuend or the subtrahend can be regrouped.         25 - 13 =          Regrouping the minuend (two examples). Taking from a multiple of ten or taking to a multiple of ten.	
	$ \begin{array}{c} 25 \\ = 7 + 5 \\ = 12 \end{array} $	5 5 5 5
	25 = 10 + 2 = 12 $(23 - 13) 2$	4 x 5 is double 2 x 5. I can show it as an array and as a linear model.
	Regrouping the subtrahend – normally to a multiple of ten. $ \begin{array}{c} 13 \\ 25 - 5 - 8 \\ = 20 - 8 \\ = 12 \end{array} $	Year 2 Numbers to 100

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: UNITISING	Core concept: CONSERVATION
	Core skill: FINDING COMPLEMENTS / REORDERING	Core skill: REGROUPING
	Adding three, one-digit numbers such as $7 + 5 + 6$ or $6 + 7 + 4$ and drawing out the reasons why pupils may wish to reorder the numbers. Focus upon a range of strategies used.	Applying understanding of benchmark numbers. Think 5 for multiplication and division $6 \times 5 = 5 \times 5 + 5 \times 1$ 5 x 5 and one more group of
	Core skill: APPLYING THE INVERSE Think addition to solve subtraction	5 is equal to 6
	Tens Ones - Ones =	
		Think 10 for multiplication and division 8 x 5 = 10 x 5 - 2 x 5
	$\begin{array}{c} 4 \\ 27 \\ 4 \\ 27 \\ 4 \\ 4 \\ 1 \\ 27 \\ 4 \\ 4 \\ 1 \\ 27 \\ 4 \\ 1 \\ 27 \\ 4 \\ 1 \\ 27 \\ 1 \\ 27 \\ 1 \\ 27 \\ 27 \\ 27 \\ $	8 x 5 is two groups of 5 fewer than 10 x 5.
	I know 4 and 3 makes 7 so 4 and 23 make 27.	Year 2 Numbers to 10



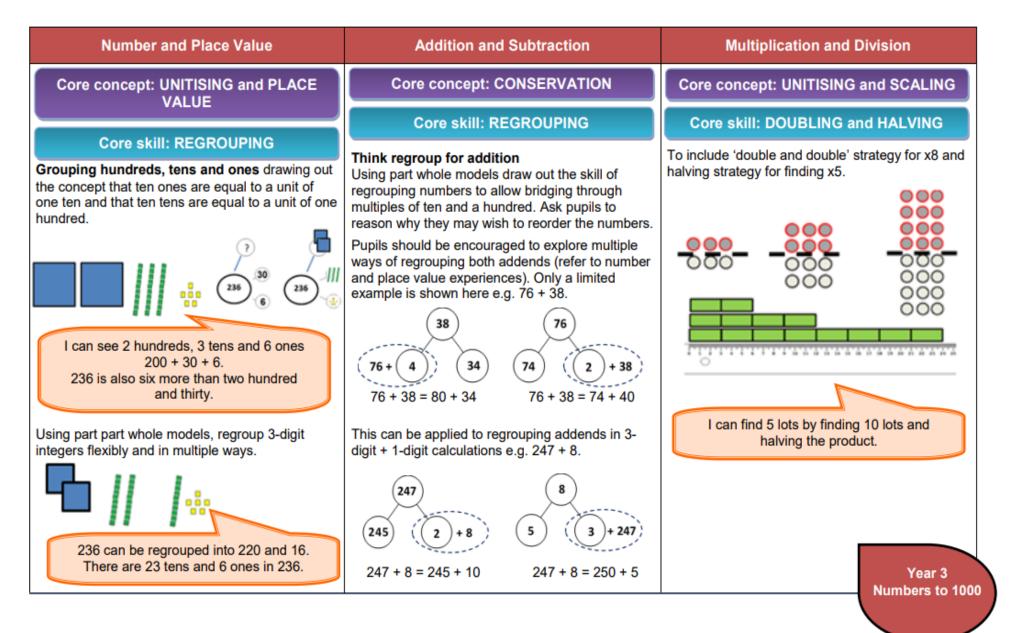


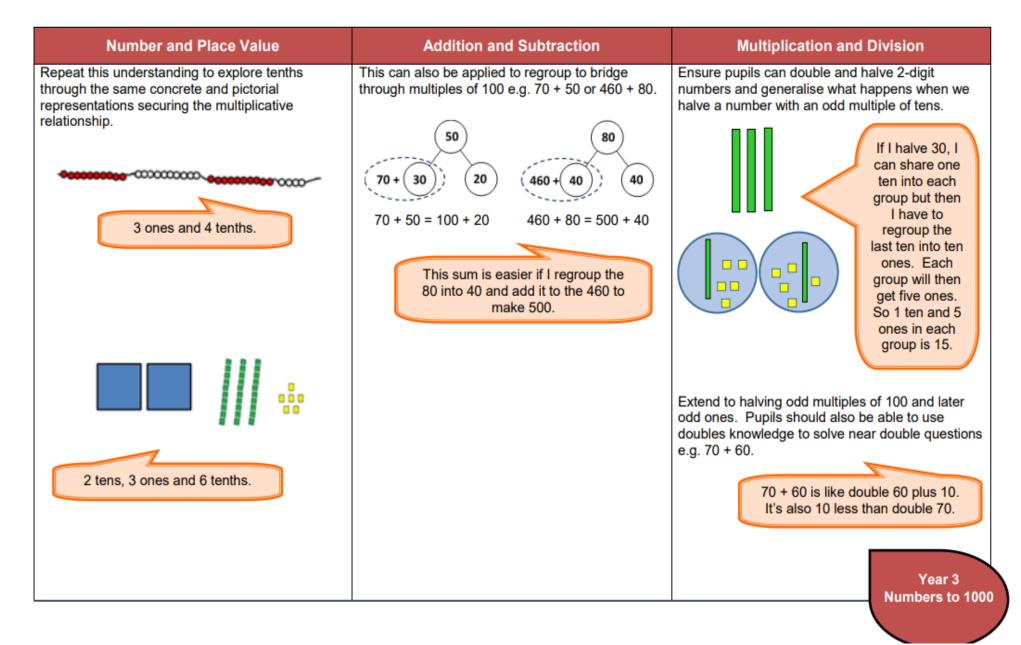


#### Key Stage 1 Examples

'Think 10' Regroup	Compensation	Double and near double facts
5+6 7+4 9+7 7+6 8+7 7+5	2+9 12+9 9+72 2+19 19+42 42+39	3 + 3 30 + 30 32 + 32 3 + 4 30 + 40
2 + 18 4 + 18 8 + 19 47 + 6 68 + 7 9 + 87	5+8 15+8 65+8 18+5 55+18 48+35	6 - 3 60 - 30 64 - 32 7 - 3 70 - 40
13 - 8 27 - 8 53 - 6 68 = 7 73 + - = 89	12 - 9 22 - 9 52 - 9 52 - 19 92 - 19 92 - 39	6 - = 3 60 - = 30 64 - =
	12 - 8 22 - 8 52 - 8 52 - 18 92 - 18 92 - 48	- 3 = 64
	48 + 🗌 = 92 8 + 🗌 = 52	
Examples from 2016 KS1 Paper 1 and Sample	Examples from 2016 KS1 Paper 1 and Sample	Find two ways of solving this: 70 - 0 = 0
Arithmetic Paper	Arithmetic Paper	
8+6= 5+7= 12-7= 46+7=	8 + 6 = 52 + 7 = 28 + = 35 69 + 11 =	Examples from 2016 KS1 Paper 1 and Sample
8 + 5 + 4 = 55 + 17 = 71 - 14 =	55 + 17 = 39 - 8 = 43 + 38 = 70 - 18 =	Arithmetic Paper
86 - 21 = 65 + = 93		12 - 7 = 🗌 🗌 + 5 = 9 50 - 🗌 = 20
	Rebalancing - Equal sum	
(Think Addition) for subtraction	12+9 9+72 24+19 15+42 44+37	Make links to doubling and halving
'Think Addition' for subtraction	5+8 15+8 65+7 18+6 55+15	3+3=2x3 2x30 2x3+1
8-5 9-6 6-2 80-50 19-6 60-20	48 + 35	6 ÷ 2 60 ÷ 2
	Examples from 2016 KS1 Paper 1 and Sample	Ensure that pupils can halve odd multiples of ten
Examples from 2016 KS1 Paper 1 and Sample	Arithmetic Paper	$50 \div 2 =$
Arithmetic Paper	8+6= 4+5+6= 69+11=	00.2-
	55 + 17 = 36 + 24 = 43 + 38 =	
17 - 6 = 39 - 8 = 50 - = 20 56 - = 51	8 + 5 + 4 =	Examples from 2016 KS1 Paper 1 and Sample
	Rebalancing - Equal difference	Arithmetic Paper
	32 - 7 25 - 8 55 - 7 55 - 17 92 - 19	$3 \times 2 = 2 \times 0 = 2$
Reordering and finding complements	97 - 43 48 + - = 92 8 + - = 55	8 ÷ 2 = 1/2 of 16 = 1/2 of 30 =
5+4+5 2+3+8 2+4+6 6+3+7		12 ÷ 2 =
36 + 5 + 4 54 + 26	Examples from 2016 KS1 Paper 1 and Sample	
	Arithmetic Paper	
	12 - 7 = 28 + = 35 71 - 14 = 20 20 21 = 72 10 = 72	
Examples from 2016 KS1 Paper 1 and Sample	39 - 8 = □ 86 - 21 = □ 70 - 18 = □	
Arithmetic Paper	65 + 🗌 = 93	
4 + 5 + 6 = 36 + 24 = 69 + 11 =	Reordering and multi-strategy	
	Examples from 2016 KS1 Paper 1 and Sample	
	Arithmetic Paper	
	8 + 5 + 4 =	
L	1	1

Key facts		
Year One Recall	Year Two Recall	
<ul> <li>Number bonds within 10 including a + b + c = d, the effect of adding zero and missing number calculations</li> </ul>	Addition and subtraction facts to 20	
• Reordering to find tens and some more e.g. 4 + 5 + 6 =	<ul> <li>Multiplication and division facts 2, 5 and 10 x tables</li> <li>Multiplication facts for 3 x tables</li> </ul>	
<ul> <li>Doubles within 10 including subtraction e.g. 6 - 3 = 3 and missing numbers e.g. 6 -</li></ul>	<ul> <li>Number of minutes in an hour; number of hours in a day</li> </ul>	
Structured subitisation on tens frame to 20	<ul><li>Coin recognition up to £2</li><li>Doubles to 20</li></ul>	



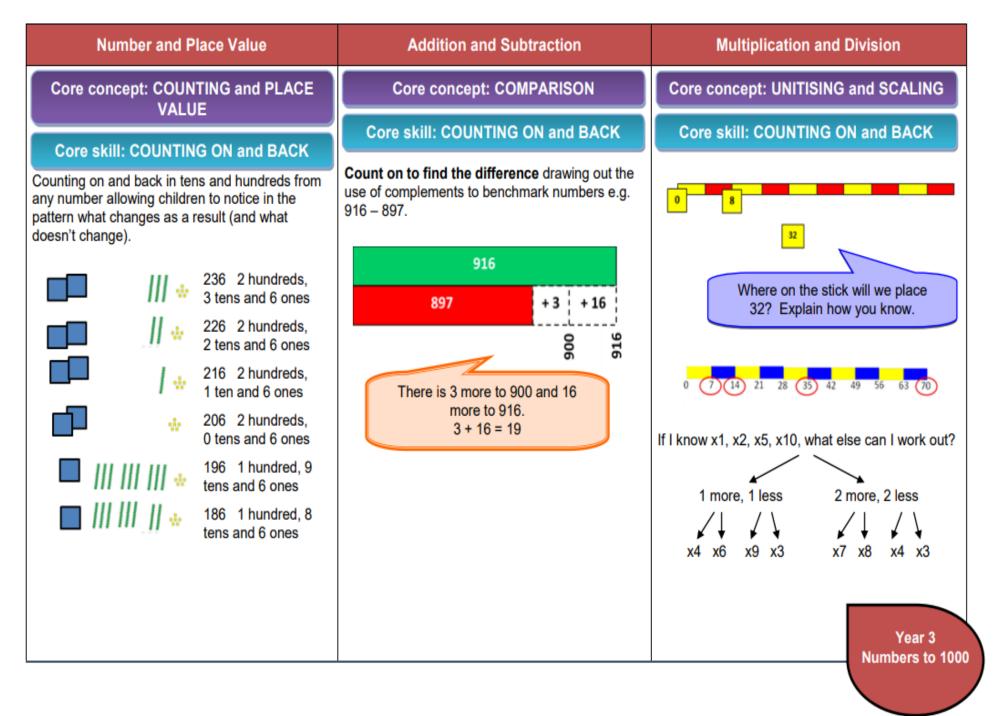


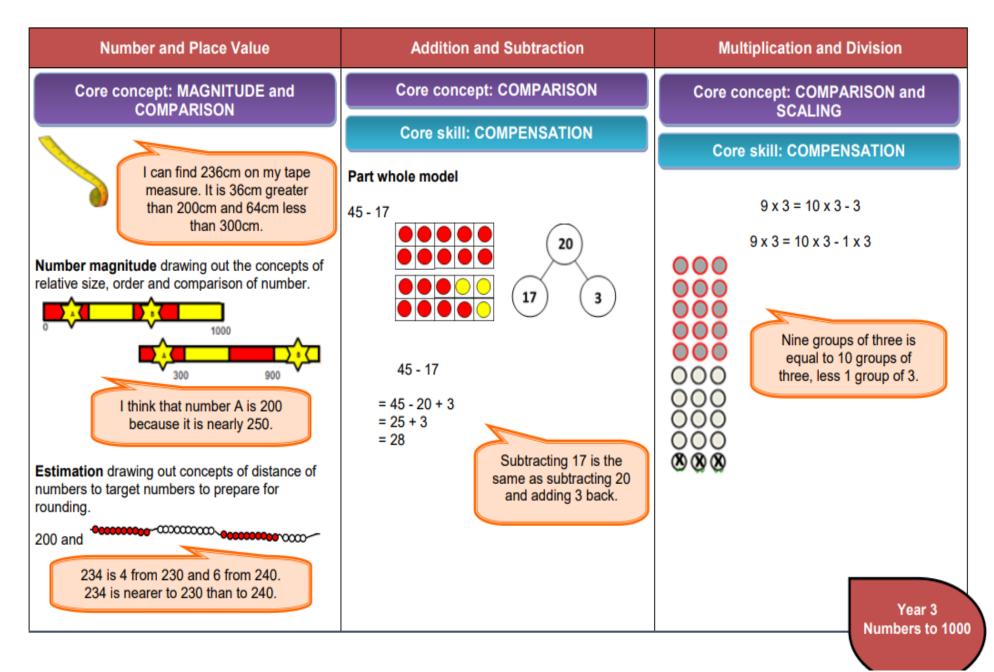
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: UNITISING	Core concept: CONSERVATION
	Core skill: REGROUPING	Core skill: REGROUPING
	Think Regroup for subtraction drawing out the skill of regrouping either the minuend or subtrahend including when bridging through multiples of ten and a hundred e.g. $25 - 13$ . Regrouping the minuend I regrouped the 25 into 20 and 5. Then I took the 13 from the 20 and that left 7. Now I add that to the 5 which makes 12.	Think 5x fact Application of the distributive law. Regrouping the multiplier (number of groups) e.g. 8 x 6. 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6
	25 23 - 13 2 I regrouped the 25 into 23 and 2. Then I took the 13 from the 23 and that left 10. Now I add that to the 2 which makes 12.	8 x 6 = 5 x 6 + 3 x 6 I can use my 5 fact to solve 8 x 6. I know 5 x 6 is 30 and 3 x 6 is 18. So 8 x 6 is 48. Year 3 Numbers to 100

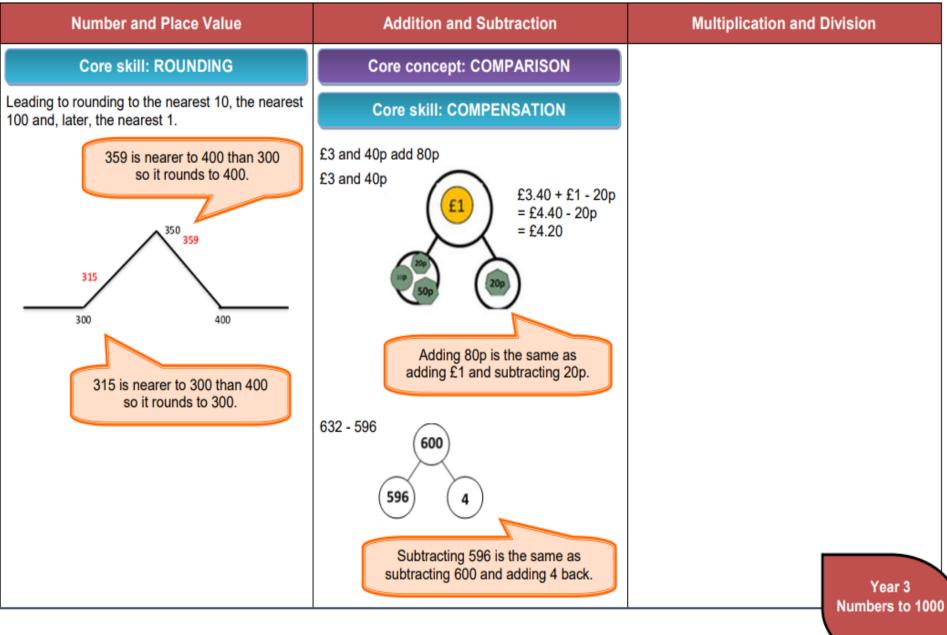
Number and Place Value	Addition and Subtraction	Multiplication and Division
Number and Place Value	Addition and Subtraction Regrouping the subtrahend	Multiplication and DivisionRegrouping the multiplicand (number in the group e.g. 8 x 6. $\bullet \bullet $
		12 x 5 = 10 x 5 + 2 x 5 Numbers to 10

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Then applied to HTO - O and HTO - TO.	
	For example, 540 - 70.	
	Regrouping the minuend 540 540	
	(500 - 70) 40 440 (100 - 70)	
	Regrouping the subtrahend	
	Apply to contexts of measures such as money and time e.g. £3 and 40p subtract 60p.	
	I can regroup the 60p into 40p and 20p. First, I take the 40p away. That gets me to £3. Next, I take the 20p away, which is £2 and 80p.	
	Core concept: UNITISING	
	Core skill: FINDING COMPLEMENTS / REORDERING	
	Reordering and finding complements	
	Adding three or more numbers. Draw out reasons	
	why children may wish to reorder the numbers. Focus upon the range of strategies used.	
	6 + 9 + 4 + 5 + 1 =	
	75 + 95 + 25 =	Year 3
	1.5 + 3 + 0.5 =	Numbers to 1

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core skill: APPLYING THE INVERSE	Core skill: APPLYING THE INVERSE
	Think addition for subtraction	Think multiplication for division e.g. 24 ÷ 8
	$\begin{array}{c} 13 \\ 45 \\ 13 \\ 13 + = 45 \end{array}$	By grouping 24 8 ? 8 I can see that 3 groups of 8 equal 24. $3 \times 8 = 24, 24 \div 8 = 3$
	285 ?	By sharing
	<b>329</b> 329 - 285 = □ so 285 + □ = 329	$\begin{bmatrix} I \text{ can see that 8} \\ groups of 3 \text{ equal 24.} \\ 8 \times 3 = 24, 24 \div 8 = 3 \\ \hline 24 \\ \hline 24 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 24 \\ \hline 1 \\ 1 \\$
		Leading to fractional understanding – see Year 2 progression and extend into further fractions. Year 3 Numbers to 1000

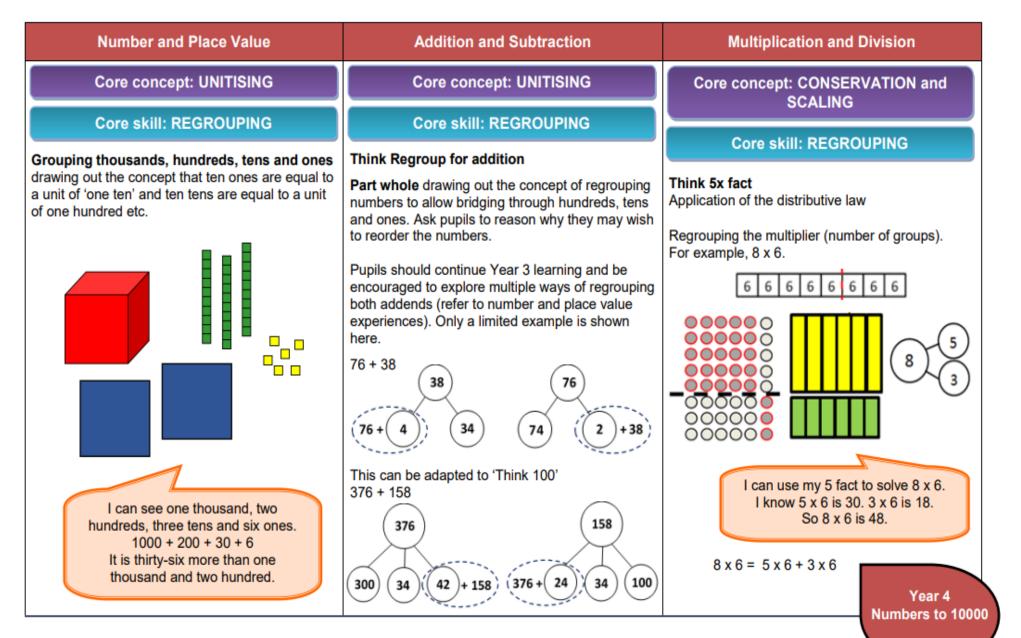


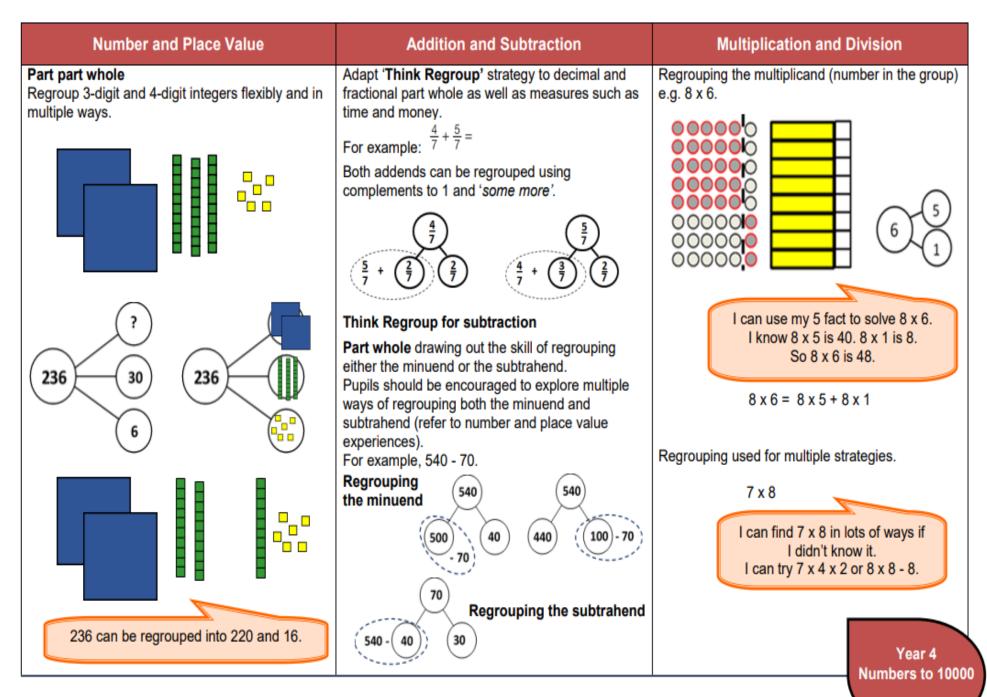


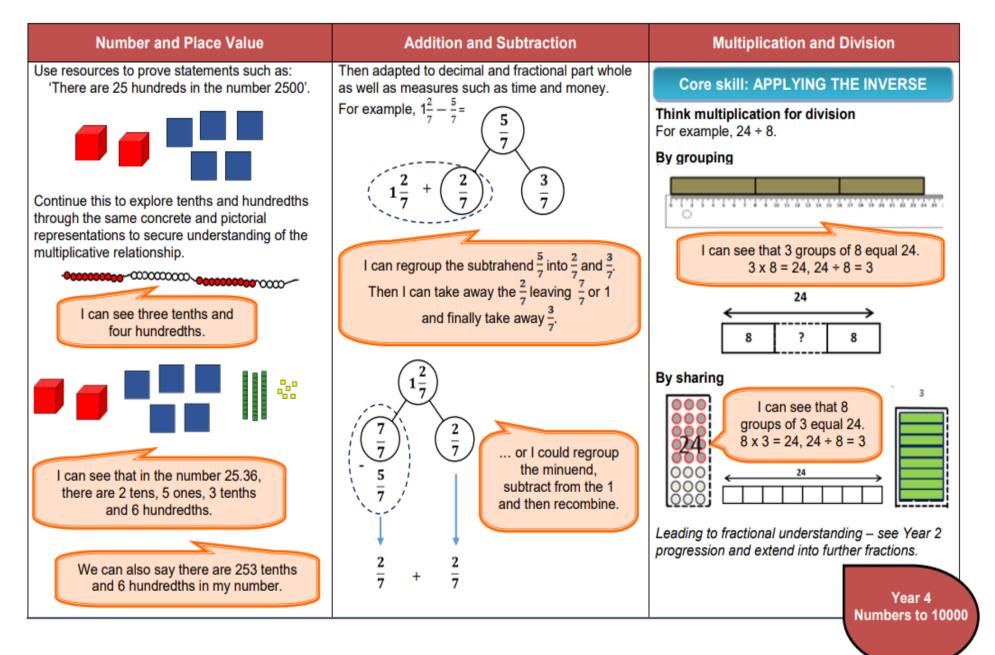


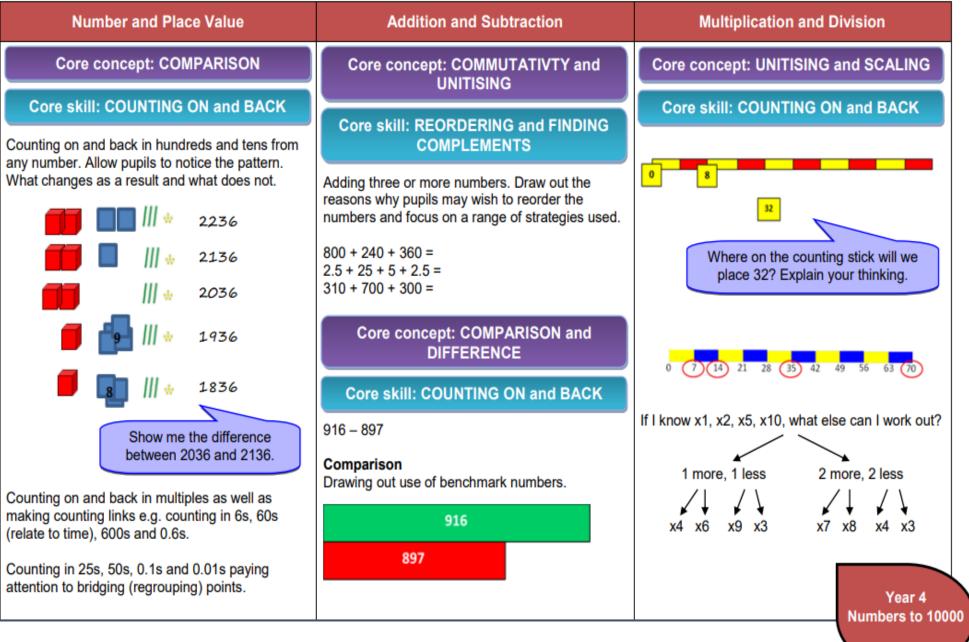
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION and SUM	Core concept: PLACE VALUE and SCALING
	Core skill: REBALANCING	Core skill: x ÷ BY 10
	Equal sum drawing out the idea of equality when we rebalance the numbers in an addition calculation e.g. 7 + 5. I can move 3 beads from the 5 to the 7 and the sum will stay the same. Now I have 10 + 2.	Check pupils understand the concept of multiplying and dividing by 1 and 0. Place value drawing out the implications of multiplying and dividing by ten and 100 on 2-digit numbers e.g. 23 x 10.
	e.g. 52 + 37 52 + 37 I move 2 beads from the 52 and give them to the 37. Now I can solve 50 + 39. It's easier.	23 groups of ten. 20 groups of ten is equal to 200. 3 groups of ten is equal to 30. 23 groups of 10 is equal to 230. Year 3 Numbers to 100

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: COMPARISON	🗆 = 3 x 40
	Core skill: REBALANCING	
	<b>Equal difference using comparison</b> drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend maintains the difference between the numbers.	
	25 13 13 10 10 10 10 10 10 10 10	3 groups of 4 tens is equal to 12 tens. 12 tens is 120. So 120 = 3 x 40 □ = 120 ÷ 3
	18 I can add 5 to each number and the difference will remain equal.	120 is 12 tens. 12 tens divided into 3 groups is equal to 4 tens. 4 tens is 40. So 40 = 120 ÷ 3.
	If I wanted to solve 21 - 16, I can take 1 from each number and solve it as 20 - 15. That is an easier calculation.	Year 3 Numbers to 1





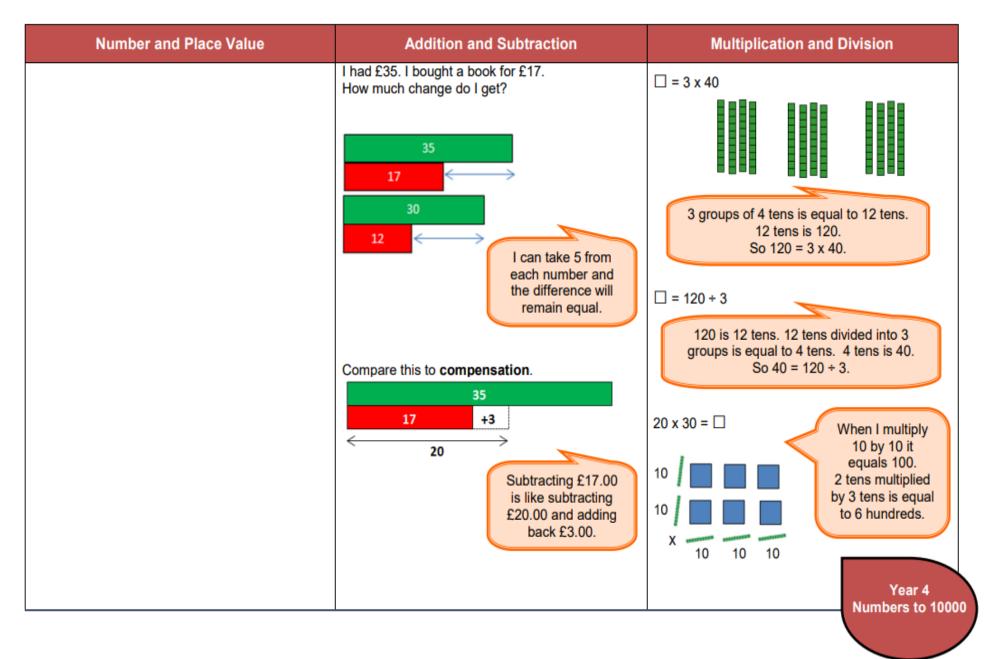




Number and Place Value	Addition and Subtraction	Multiplication and Division
Core concept: MAGNITUDE	Core concept: COMPARISON, SUM and DIFFERENCE	Core concept: COMPARISON and SCALING
Number magnitude drawing out the concepts of relative size, order and comparison of number.	Core skill: COMPENSATION and REBALANCE	Core skill: COMPENSATION
	Equal sum drawing out the concept of equality when rebalancing the numbers in an addition calculation.	3 x 9 = 3 x 10 - 3
The number couldn't bebecause It could be because	255 + 49 is easier if I take one from the 255 and give it to the 49. My sum stays equal. Then my sum becomes 254 + 50 = 304.	Nine groups of three is equal to
0.5 Number estimation (using scales) should be applied to different scales of measures. This should include those with negative and dial scales.	<b>Compensation</b> with the same calculation supports pupils' multi-strategy approach. Pupils can begin to evaluate strategies.	three is equal to ten groups of three, less 1 group of 3.
Core skill: ROUNDING	compensation because adding 49 is like adding 50 and taking one away. Now my sum looks like this:	
Leading to rounding to the nearest 10, 100, 1000, hour and £1 etc.	255 + 50 - 1 = 304.	
		Year 4 Numbers to 1000

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Rebalancing	Core concept: CONSERVATION and SCALING
	It is 1.44pm. What time will it be in 50 minutes?	Core skill: REARRANGING Children should explore rearranging arrays to
	1 hour 44 minutes + 50 minutes = 1 hour 34 minutes + 60 minutes = 2 hours and 34 minutes = 2.34pm If I give 10 minutes from the 1 hour 44 to the 50 minutes then I can add on 1 hour. Compensation It is 1.44pm. What time will it be in 50 minutes? Adding 50 minutes is like adding one hour and taking away 10 minutes.	simplify multiplications but conserve the area. 4 x 16 = Can become 8 x 8 Can bec

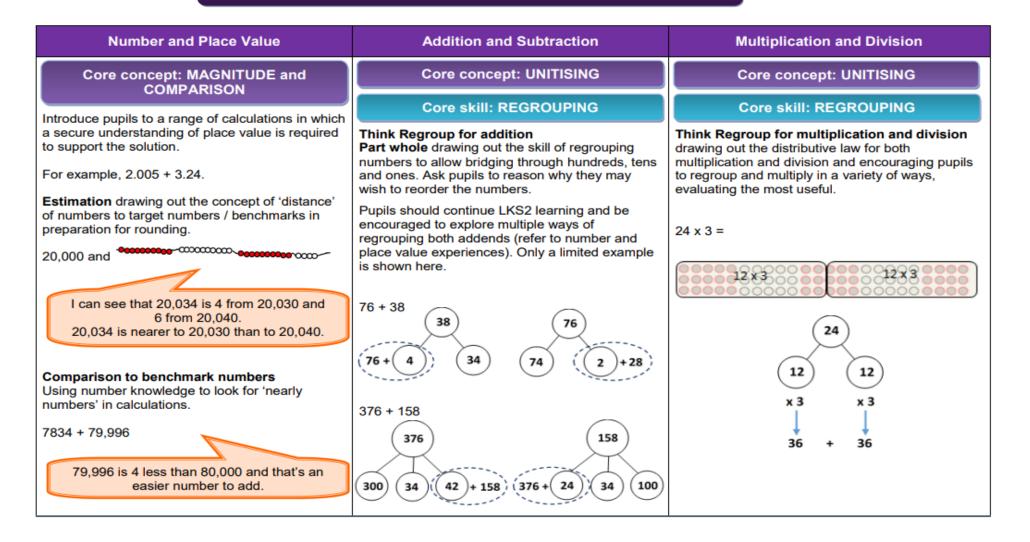
Number and Place Value	Addition and Subtraction	Multiplication and Division
	Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend	Core concept: PLACE VALUE
	maintains the difference between the numbers.	Core skill: x ÷ BY 10, 100
	187 56 191 191 187 I can add 4 to each number and the difference will remain equal.	Check pupils understand the concept of multiplying and dividing by 1 and 0. Place value drawing out the implications of multiplying and dividing by 10 and 100 on 2-digit numbers e.g. 23 x 10.
	181 I can take 6 from each number and the difference will remain equal. Which strategy did you prefer? Explain why.	10 23 groups of ten. 20 groups of ten is equal to 200. 3 groups of ten is equal to 30. 23 groups of 10 is equal to 230.
		Year 4 Numbers to 1



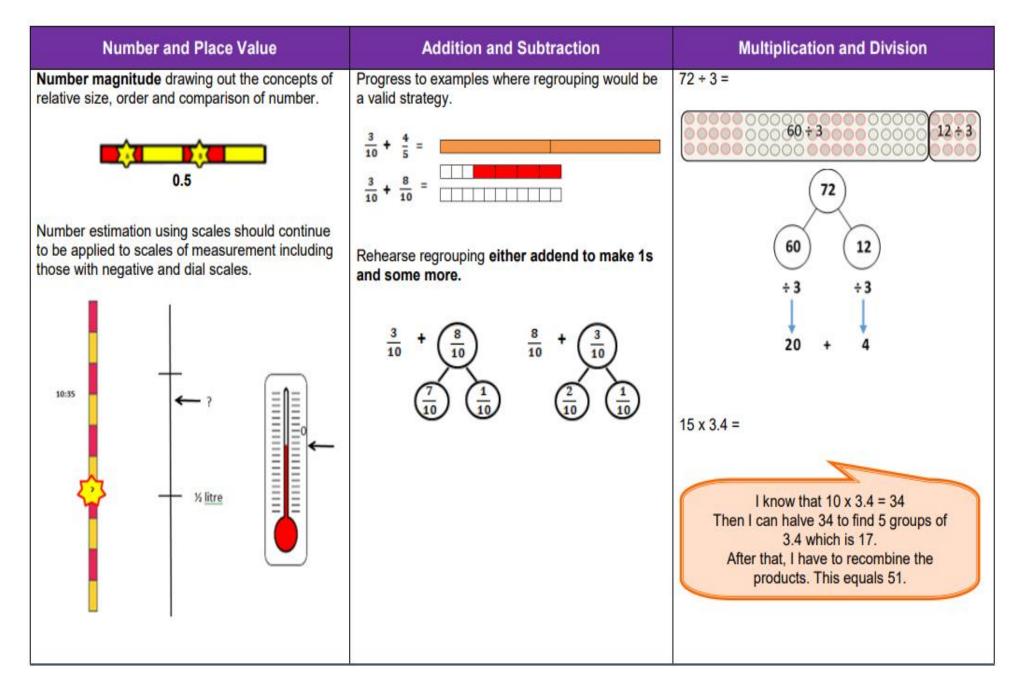
#### Lower KS2 examples

'Think Regroup' for addition	Re-ordering and finding complements T	Think multiplication
Think 10		35 ÷ 5 72 ÷ 4 99 ÷ 6 240 ÷ 12
37 + 45 68 + 23 29 + 75	8+6+2+3+4 3+5+7+5+4 6	$560 \div 3$ 210 ÷ 7 540 ÷ 9 500 ÷ 4
76 + 27 $55 + 16$ $42 + 38$	1 + 4 + 6 +7 + 9 30 + 50 + 70	_ ÷ 3 = 8 3 _ ÷ 5 = 6
537 + 8 727 + 5 213 + 18 146 + 37	25 + 50 + 5 75 + 40 + 20 + 25	
36 - = 29 $56 - 2 = 33$ $7 - 45 = 32$		X and ÷ 10, 100 and 1000
	$400 \pm 547 \pm 600$ $700 \pm 240 \pm 200$	4 x 30 9 x 30 70 x 70 60 x 50
Think 100	$750 \pm 400 \pm 250$	300 x 4 800 x 7 9 x 800 6 x 400
290 + 13 370 + 50 580 + 73	3	3 x 2000 4000 x 6 8 x 7000 9 x 8000
270 + 51 67 + 350 860 + 69		500 ÷ 10 400 ÷ 5 600 ÷ 2 240 ÷ 4
86 + 770 680 + 63		120 ÷ 🗌 = 12 365cm = 🗌 m 750mm = 🗌 cm
	Compensation	
Think 1000		Double and near double facts
4900 + 500 4800 + 260 6900 + 430		7 x 20 3 x 38 9 x 200 11 x 4
3200 + 910 230 + 7900 570 + 8500		16 x 20 18 x 2000
3700 + 370 3622 + 500		80 ÷ 4 160 ÷ 4 1600 ÷ 4 2400 ÷ 4
Think 4	43 - 18 94 - 37 54 - 29 77 - 9 82 - 23	Think 5 / Think 40 for multiplication
Think 1		Think 5 / Think 10 for multiplication
$2.7 + 1.4$ $2\frac{8}{10} + \frac{3}{10} = 6.5 + 5.6$		28 x 5 16 x 8 23 x 9 92 x 8 52 x 4 13 x 21 34 x 19 123 x 4 214 x 6 9 x 234
$1\frac{7}{5} + 1\frac{5}{5}$	200 100 100 100	13 x 21 34 x 19 123 x 4 214 x 6 9 x 234 11 x 314 21 x 400 400 x 38
8 8	Rebalancing - Equal sum	11 x 314 21 x 400 400 x 36
	45 + 27 26 + 39 78 + 18 65 + 27	
'Think Regroup' for subtraction	73 + 39 84 + 47 42 + 97 116 + 35	
Think 10	368 + 123 404 + 198 356 + 427 528 + 298	
97 - 8 74 - 7 53 - 5 63 - 37	3.7 + 1.9 7.6 + 4.7 1.9 + 5.8	
77 - 32 84 - 26 57 - 28 256 - 37	Rebalancing - Equal difference	
25 + = 85 163 + = 363 426 + 22 = 668	75 - 28 56 - 29 78 - 38 55 - 27	
	83 - 21 75 - 12 95 - 42 67 - 51	
230 - 70 660 - 82 420 - 77 950 - 147	912 - 797 837 - 498 711 - 467 628 - 198	
230 - 70 000 - 62 420 - 77 950 - 147	482 - 302 729 - 404 548 - 202 637- 203	
Think 1	6.4 - 3.9 6.6 - 3.2 7.7 - 4.8 $1\frac{2}{7}-\frac{5}{7}$	
1.3-0.6 $1\frac{4}{8} - 1\frac{5}{8}$ 3.4-2.7 $2\frac{1}{3} - 1\frac{2}{3}$		
1.3 - 0.6 18 18 3.4 - 2.7 3 3	Counting on to subtract	
	315 - 298 412 - 396 917 - 898 611 - 598	

#### Years 5 and 6



Number and Place Value	Addition and Subtraction	Multiplication and Division
Rounding Round 136,521 to the nearest 100, 1000 and 10,000. Rounding as estimation for multiplication and division. 688 x 79 =	Then adapted to decimal and fractional part whole as well as measures such as time and money. For example, $\frac{4}{7} + \frac{5}{7} =$ Here both addends can be regrouped using complements to 1 and <i>some more</i> .	24 x 3 =
688 rounds to 700 and 79 rounds to 80. The calculation 688 x 79 is close to 700 x 80, which is 56,000.	$ \begin{array}{c} \begin{pmatrix} 4\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7\\7$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
789 + 79 =	Extend into UKS2 by converting fractions into equivalents with common denominators. Beginning with conversions where no regrouping is required.	
789 rounds to 800 and 79 rounds to 80. The calculation 789 ÷ 80 is close to 800 ÷ 80, which equals 10.	For example: $\frac{2}{10} + \frac{2}{5} =$ $\frac{2}{10} + \frac{4}{10} =$	



Number and Place Value	Addition and Subtraction	Multiplication and Division
	Think regroup for subtraction	
	Part whole drawing out the skill of regrouping either the minuend or the subtrahend.	
	Pupils should be encouraged to explore multiple ways of regrouping both the <b>minuend</b> and <b>subtrahend</b> (refer to number and place value experiences).	
	For example, 540 – 70	
	Regrouping the minuend	
	540 540 500 - 70 40 440 (100-70)	
	Regrouping the subtrahend	
	70 (540 - 40); 30	

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Then adapted to decimal and fractional part whole as well as measures such as time and money. For example, $1\frac{2}{7} - \frac{5}{7} = 5$ 7 $1\frac{2}{7} + 2$ $1\frac{2}{7} + 2$ 7 I can regroup the subtrahend $\frac{5}{7}$ into $\frac{2}{7}$ and $\frac{3}{7}$ . Then I can take away the $\frac{2}{7}$ leaving $\frac{7}{7}$ or 1 and finally take away $\frac{3}{7}$ .	
	$\begin{array}{c} 1 & 7 \\ 1 & 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7 \\ 7$	

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Extend into UKS2 by converting fractions into equivalents with common denominators.	
	Beginning with conversions where no regrouping is required. For example, $\frac{2}{10} - \frac{1}{20} =$ Progress to examples where regrouping would be a valid strategy. For example, $1\frac{3}{10} - \frac{4}{5} =$	
	Pupils will have to know that $\frac{4}{5} = \frac{8}{10}$ before they can solve the calculation.	
	Then they could regroup either the subtrahend or the minuend.	
	For example, $1\frac{3}{10} - \frac{8}{10} =$	
	$1\frac{3}{10}$ - $\frac{8}{10}$ Partitioning the subtrahend $\frac{3}{10}$ $\frac{5}{10}$ $(\frac{3}{10})$	
	Partitioning the minuend $\begin{pmatrix} 1\\ 1\\ \frac{8}{10} \end{pmatrix}$	
	$\frac{10}{10}$ + $\frac{3}{10}$	

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION	Core concept: CONSERVATION
	Reordering and finding complements across a range of numbers.	Core skill: REARRANGING
	For example: $\begin{array}{c} 47 + 603 & 0.45 + 1.63 \\ 0.15 + 1.85 & \pounds 3.99 + \pounds 7.80 + \pounds 2.01 \end{array}$ Two decimal numbers add together to make a total of 1. One number is 0.0006. What is the other number?	Factorisation drawing on the associative law for multiplication and related division facts. For example, $24 \times 3 = 12 \times 3 \times 2$ 12 × 3 Two and twelve are factors of 24 and I find it easier to calculate 12 x 3 first and then double it. Doubling and halving $12 \times 2.5 =$ $12 \times 2.5 = 6 \times 5$ . I halved the 12 and doubled the 2.5 to make the calculation easier. $16 \times 6 \frac{1}{4} =$ $16 \times 6 \frac{1}{4} = 8 \times 12 \frac{1}{2} = 4 \times 25 = 100$ I can make this easier for me by doubling and doubling again the 6 $\frac{1}{4}$ . This means I have to halve and halve again the 16 to maintain the area. Now I get $4 \times 25 = 100$ . Application to KS2 example (Q11 paper 1 2016): $71 \times 8 = 142 \times 4$ $= 284 \times 2$

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Core concept: CONSERVATION and COMPARISON	Halving and halving for division Once pupils are confident with the 'halve and double' strategy for multiplication, they will try to
	Core skill: REBALANCING	apply it to division and will need to understand why their answers do not make sense. Stress again the
	<b>Equal sum</b> drawing out the concept of equality when rebalancing the numbers in an addition calculation.	importance of estimation. Investigate the principle of halving and halving with pupils.
		- 72 ÷ 4 = (72 ÷ 2) ÷ 2
	Pupils use bead strings to demonstrate that: 7 + 5 = 10 + 2 Apply concept to range of numbers and missing	When I am dividing by 4, I like to halve the number and halve it again.
	number problems. For example, $24 + \Box = 30 + 3$ .	
	See Year 3 and 4 examplesThese should include rehearsal using calculationssuch as:39 + 52345 + 198	This strategy is best explored through practical contexts so pupils can clearly see that even though the dividend and the divisor are changing the quotient remains constant.
	0.39 + 6.54 5.1 + 2.7 = 🗆 + 4.8	

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Ensure pupils are secure with the concept of equal sum before considering questions such as:	For example, If I shared 12 cookies among 4 children each child would get 3 cookies.
	7834 + 79,996 79,996 is 4 away from 80,000. I can rebalance the sum by taking 4 from 7834 and giving it to the 79,996. Now I have 80,000 + 7,830 = 87,830.	12 ÷ 4 = 3 I can also see that 6 cookies shared between 2 people would give the same group size. The size of the group hasn't changed. So 12 ÷ 4 can be changed into 6 ÷ 2.
	Compensation with the same calculation supports pupil's multi-strategy approach. Pupils can continue to <i>evaluate</i> strategies. 7834 + 79,996 Adding 79,996 is like adding 80,000 and subtracting 4. I can do 80,000 + 7834 - 4 = 87,830	As I am trying to find out the group size, I can also see that 3 ÷ 1 gives me the group size. So 12 ÷ 4 can be thought of as 6 ÷ 3 and 3 ÷ 1. I can see all of these in the array.
	Improve multi-strategy approaches by asking for two different ways of solving calculations such as: $\Box = 5,756 + 8,643$ 16.98 + 23.214 = $\Box$	Applying this conceptual understanding to larger numbers encourages playfulness with division. $364 \div 16 =$ $182 \div 8 =$ $91 \div 4 =$ $45.5 \div 2 =$ 22.75 I saw that I could halve both the dividend and the divisor, so I did to see if it made it easier. Then I realised that I could halve them again and again.
		Apply core concepts in the context of fractions.

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Equal difference using comparison drawing out the concept that adding or subtracting the same quantity from both the subtrahend and minuend	Core concept: UNITISING
	will maintain the difference between the numbers.	Core skill: REGROUPING
	187 56 I can add 4 or take 6 away	<b>Division of fractions by integers</b> drawing out the concept of multiple groups of the numerator before teaching a rule. Ensure that pupils always refer to the whole.
	60 from each of the numbers and the difference will be the same.	I know that 12 ÷ 3 can be thought of as 'If I share 12 equally between 3 groups, how many in each group?' $\frac{6}{7}$ ÷ 3 can be thought of as, 'If I share $\frac{6}{7}$ equally between 3 groups, how many in each group?'
	<b>Remember</b> to rehearse simple calculations such as 367 - 9 before applying to a range of numbers.	$\frac{6}{7} \div 3$
	132,457 - 11,999 =	
	11,999 is nearly 12,000. If I add one to each number the difference will stay equal. Now my calculation is 132,458 - 12,000 =	$\begin{array}{c c} \hline \\ \hline $

Number and Place Value	Addition and Subtraction	Multiplication and Division
	Use a range of examples. = 4 - 1.15 4 1.15 3.85 1 It is easier if I subtract 0.15 from each number. The difference will stay the same. Now my calculation is 3.85 - 1 = Compare this to compensation 132,457 - 11,999 = Subtracting 11,999 is like subtracting 12,000 and then adding 1. Now my calculation is 132,457 - 12,000 + 1 =	Progress to dividing fractions in which the fraction needs converting. $\frac{3}{4} \div 2$ $\frac{3}{8} \times \frac{3}{8}$ $\frac{3}{8} \times \frac{3}{8}$ $\frac{3}{8} \div 2$ $\frac{3}{8} \times \frac{3}{8} \times \frac{3}{8}$ $\frac{3}{8} \div 2$

Number and Place Value	Addition and Subtraction	Multiplication and Division
		Multiplication of fractions by fractions
		Equal groups
		I know that 3 x 4 could mean 3 groups of 4. So ½ x ¼ means half a group of ¼.
		1/2 x 1/4 =
		When we find half of any number, we divide it by two. The blue part has a value of $\frac{1}{4}$ . When I halve it, it makes $\frac{1}{8}$ .
		Pupils should focus upon the denominators and reason why, when multiplied, we find the product of the denominators. Once understood pupils can employ the rule.

Number and Place Value	Addition and Subtraction	Multiplication and Division
		Halve and double
		The 'halve and double' rule can be applied to fractions.
		Pupils have already secured conceptual understanding of this rule, for example:
		5 x 4 = 10 x 2 = 20 x 1
		Apply this understanding to fractions, for example: $\frac{1}{2} \times \frac{1}{4} =$
		If we double the first term and halve the second, we can transform the calculation to:
		$1 \times \frac{1}{8} = \frac{1}{8}$
		For further detail regarding the multiplication and division of fractions refer to the 'HfL Bar Modelling Progression' document.

#### Upper KS2 examples

Place Value	Compensation	Think Partition for x and ÷
937 + 100 1969 + 100 546 - 40	56 + 8 72 + 9 56 - 8 72 - 9	32 x 4 29 x 2 122 x 4 4.6 x 2
1.7 + 0.05 40 000 - 500	371 + 18 255 + 49 304 + 299	75 x 3 8.3 x 6 39 x 7
246 ÷ 1 100 x 217 0.4 ÷ 10	673 - 99 854 - 398 3720 - 996	3.3 x 7 5 x 49 4 x 198 96 x 0.3
1.68 x 100 100 x 100	0.71 + 0.09 0.56 + 0.08 0.34 - 0.09	
Examples from 2016 KS2 and Sample Papers	£1.17 + £0.39 £8.89 - £4.99	Examples from 2016 KS2 and Sample Papers
435 - 30  979 + 100  3.005 + 6.12  2.15 + 0.05		15 x 6.1 24 x 3 1.52 x 6 7,505 ÷ 5
$100 \times 412$ $0.9 \div 100$ $1.28 \times 100$ $50,000 - 500$	Examples from 2016 KS2 and Sample Papers	17 x 1½
10 x 100	468 - 9 472 - 9 15.98 + 26.314	
	12 - 6.01 15.4 - 8.88	Make links to doubling and halving
Two decimal numbers add together to equal 1 One of		50 x 28 86 x 50 500 x 70 18 x 2.5
the numbers is 0.007. What is the other number?	Rebalancing - Equal sum	86 x 2.5 160 x 35 500 x 88 1.5 x 6.6
Circle two numbers that added together make 0.25	56 + 8 72 + 9 371 + 18 255 + 49	0.5 x 120 4.5 x 2.2 15% x 346 75% x 220
0.05 0.23 0.2 0.5	304 + 267	
Circle two numbers that multiply together to equal 1	£37.67 + £3.85 563 + 397 890,488 + 4,890	Examples from 2016 KS2 and Sample Papers
million 200 2,000 5,000 50,000	229,899 + 31,321	2
		15% x 440 5 x 140 24 x 3
Write the number that is 5 less than 10 million	Examples from 2016 KS2 and Sample Papers	20% of 1500 95% of 240
Write the number that is one hundred thousand less	89,994 + 7,643 936 + 285 89,994 + 7,643	
than six million		
Bound 124 521 to the pearest 10,000, 1,000, 100	Rebalancing - Equal difference	Multiplying and dividing fractions
Round 124,531 to the nearest 10,000, 1,000, 100	85 - 18 42 - 17 88 - 43 437 - 103	Examples from 2016 KS2 and Sample Papers
Think Regroup	819 - 504 532,525 - 9897	$\frac{3}{5} \div 3$ $\frac{2}{5} \div 2$ $\frac{3}{4} \div 2$ $\frac{2}{5} \times 140$ $\frac{1}{4} \times \frac{1}{8}$
58 + 6 5 + 47 630 + 73 680 + 78	£122.56 - £87.99 9.1 - 6.7 15.3 - 5.7	5 $5$ $4$ $5$ $4$ $5$ $4$ $8$
560 + 89 8900 + 230		
74 - 7 97 - 8 320 - 50 2300 - 600	Examples from 2016 KS2 and Sample Papers	
3400 - 1700	468 - 9 472 - 9 122,456 - 11,999	
5 - 2.65 8.1 - 2.75 $1\frac{2}{5} + \frac{3}{10} = 1\frac{3}{10} - \frac{2}{5} =$	4 - 1.15 12 - 6.01	
	15.4 - 8.88 234,897 - 45,996	
£3367.40 - £1021.23		
Examples from 2016 KS2 and Sample Papers		
$\frac{4}{4} + \frac{3}{3}$ $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} - \frac{1}{2}$ $\frac{3}{2} + \frac{7}{2}$		
4-1.15 15'10 14'3 15 4 4'8=		
5,756 + 8,643 936 + 285		

#### Upper KS2 examples (continued)

Re-ordering and finding complements	x and ÷ by powers of 10
Re-ordering and finding complements           11 + 59         33 + 57         14 + 90 + 86           290 + 310         1.15 + 2.55         0.8 + 0.26           Examples from 2016 KS2 and Sample Papers         1,034 + 586         2.15 + 0.05           Circle two numbers that added together make 0.25         0.05         0.23         0.2         0.5	x and $\div$ by powers of 10         10 x 53       87 x 10       1000 x 14       100 x 8.3         100 x 0.41       30 x 3       7 x 0.3       30 x 30       30 x 70         567 $\div$ 100       36 $\div$ 10       0.5 $\div$ 10       280 $\div$ 4         5600 $\div$ 80       30 = $\Box$ $\div$ 12       270 $\div$ 9 = $\Box$ $\div$ 0.9         7 x 0.001       1.8 $\div$ 0.1       3.25 $\div$ 0.00001         Circle two numbers that multiply together to equal 10 million       200       2,000       50,000         Examples from 2016 KS2 and Sample Papers       1,320 $\div$ 12       0.9 $\div$ 10       20% of 1,800       20% of 1500       7,505 $\div$ 5         95% of 240       100 x 412       0.9 $\div$ 10       1.28 x 100       50,000 $\div$ 500       10 x 100
	Circle two numbers that multiply together to equal 1 million 200 2,000 5,000 50,000